8/28/19

Module 1: Survival Analysis (p's & q's)

Section 1: Basics

(x): person who is age x

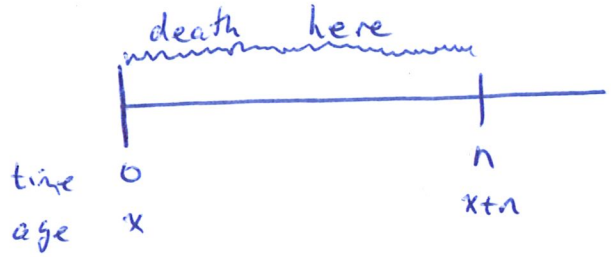
"rvr" - random variable representing

T_x = rvr the time until the death of (x)

$\text{supp}(T_x) = [0, \infty) \Rightarrow T_x$ is a continuous r.v.

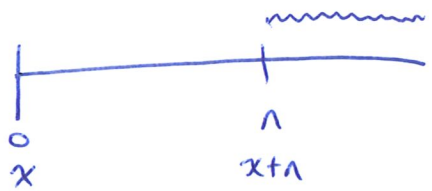
cdf: $F_{T_x}(t) = F_x(t) = Pr(T_x \leq t) \overset{\text{actuarial notation}}{=\!=\!=} {}_tq_x$

${}_nq_x = Pr(\text{(x) dies before age } x+n)$



sf: $S_{T_x}(t) = S_x(t) = Pr(T_x > t) \overset{\text{actuarial notation}}{=\!=\!=} {}_tp_x$

${}_np_x = Pr(\text{(x) survives to age } x+n)$



Examples: ${}_0p_x = 1$ ${}_0q_x = 0$

${}_\infty p_x = 0$ ${}_\infty q_x = 1$

Remarks:

1) ${}_nP_x + {}_nq_x = 1$

2) Notation $\quad p_x = {}_1p_x$

$\quad q_x = {}_1q_x$

3) Notation

${}_nP_x$ ← age

↓ duration (time value)

Factoring Survival Probabilities (p's)

${}_{k+n}P_x = Pr(\text{(x) survives to age } x+k+n)$

$= \underbrace{Pr(\text{(x) survives to age } x+k)}_{{}_kP_x} \cdot \underbrace{Pr(\text{(x+k) survives to age } x+k+n)}_{{}_nP_{x+k}}$

${}_{k+n}P_x = {}_kP_x \cdot {}_nP_{x+k} = {}_nP_x \cdot {}_kP_{x+n}$

Examples: (See next page)

Module 1 Section 1 Example 1

You are given:

i) The probability that (40) survives to age 45 is 0.9.
ii) The probability that a person who is 40 dies after age 60 is 0.7.

Determine the percentage of 45-year olds who live longer than 15 years.

Solution:

i) ${}_5P_{40} = 0.9$

ii) ${}_{20}P_{40} = 0.7$

Q: ${}_{15}P_{45} = ?$

A: ${}_{20}P_{40} = {}_5P_{40} \cdot {}_{15}P_{45}$

$\Rightarrow {}_{15}P_{45} = \frac{7}{9}$

Module 1 Section 1 Example 2

You are given:

i) The probability that (35) dies within 35 years is 0.68.
ii) The proportion of 50 year olds who survive to age 70 is 0.4.

Determine the probability that a randomly chose 35-year old dies before age 50.

Solution:

i) ${}_{35}q_{35} = .68 \Rightarrow {}_{35}p_{35} = .32$

ii) ${}_{20}p_{50} = .4$

Q: ${}_{15}q_{35} = ? = 1 - {}_{15}p_{35}$

${}_{35}p_{35} = {}_{15}p_{35} \cdot {}_{20}p_{50}$

$\Rightarrow {}_{15}p_{35} = \frac{32}{40} = 0.8$

$\therefore {}_{15}q_{35} = 0.2$